

## Symbols for the Inverse of a Function

If function  $g$  is the inverse of function  $f$ , the symbol  $f^{-1}$  is often used for the name of function  $g$ . In the highway stripe example, you can write  $f^{-1}(0.2) = 1$ ,  $f^{-1}(0.6) = 2, \dots, f^{-1}(3) = 8$ . Note that  $f^{-1}(3)$  does not mean the *reciprocal* of  $f(3)$ .

$$f^{-1}(3) = 8 \quad \text{and} \quad \frac{1}{f(3)} = \frac{1}{1} = 1, \text{ not } 8$$

The  $-1$  used with the *name* of a function means the *function* inverse, whereas the  $-1$  used with a *number*, as in  $5^{-1}$ , means the *multiplicative* inverse of that number.

## Inverse of a Function Graphically

Figure 1-5a shows a graph of the data for the highway stripe example. Note that the points seem to lie in a straight line. Connecting the points is reasonable if you assume that the crew paints continuously. The line meets the  $t$ -axis at about  $t = 0.5$  h, indicating that it takes the crew about half an hour at the beginning of the shift to redirect traffic and set up the equipment before it can start painting.

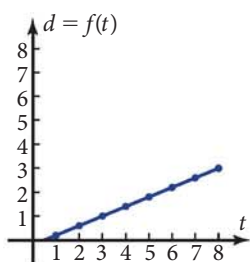


Figure 1-5a

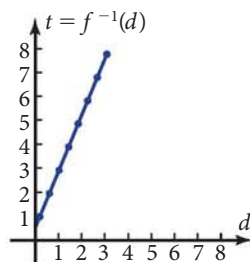


Figure 1-5b

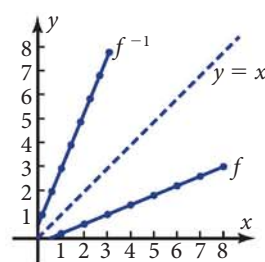


Figure 1-5c

Figure 1-5b shows the inverse function,  $t = f^{-1}(d)$ . Note that every vertical feature on the graph of  $f$  is a horizontal feature on the graph of  $f^{-1}$ , and vice versa. For instance, the graph of  $f^{-1}$  meets the *vertical* axis at 0.5.

Figure 1-5c shows both graphs on the same set of axes. In this figure,  $x$  is used for the input variable and  $y$  for the output variable. Keep in mind that  $x$  for function  $f$  represents hours and  $x$  for function  $f^{-1}$  represents miles. The graphs are **reflections** of each other across the line whose equation is  $y = x$ .

## Inverse of a Function Algebraically

In the highway stripe example, the linear function that fits the graph of function  $f$  in Figure 1-5c is

$$y = 0.4(x - 0.5) \quad \text{or, equivalently,} \quad y = 0.4x - 0.2 \quad \begin{array}{l} \text{slope} = 0.4, \\ \text{x-intercept} = 0.5 \end{array}$$